2024/09/30 - Transform Techniques - Week 04

Recall : $\quad L \cdot T \left(t f(t) \right) = - F'(s) \quad$ where $\quad F(s) = L \cdot T \left(f(t) \right)$

$$L \cdot T \cdot \left(\frac{f(t)}{t} \right) = \int_s^\infty f(\bar{\sigma}) \, d\bar{\sigma}$$

Set $\quad g(t) = \dfrac{f(t)}{t} \quad \Rightarrow \quad t \cdot g(t) = f(t)$

From ① : $\quad L \left(t \cdot g(t) \right) = - G'(s)$

$\because \quad F(s) = - G'(s) \quad \Rightarrow \quad G(s) = - \int_a^s F(\tau) \, d\tau$

$\because \quad \lim_{s \to \infty} G(s) = 0 \quad \Rightarrow \quad \int_a^\infty F(\tau) \, d\tau = 0 \quad \Rightarrow \quad G(s) = 0 - \int_a^s F(\tau) d\tau$

$$= \left(\int_a^\infty - \int_a^s \right) F(\tau) d\tau$$

$$= \int_s^\infty F(\tau) \, d\tau$$

Show that $\mathcal{L}\left(\int_0^s f(\tau)\, d\tau \right) = \dfrac{F(s)}{s}$

Set $g(t) = \int_0^t f(\tau) = d\tau \Rightarrow g'(t) = f(t)$

$\downarrow$ laplace transform of derivative

$\therefore\ s \cdot G(s) = F(s)$

$\Rightarrow G(s) = \dfrac{F(s)}{s}$

Problems

① Find $f(t)$ such that $F(s) = \dfrac{4}{(s-1)^3}$

② Find $L \cdot T \left(\int_0^t \sin(az)\, dz \right)$, find $L \cdot T \left(e^{at} \sin bt \right)$