$\begin{array}{c} A \quad is \quad r.e., \quad A \leq_m B \\ \hline \\ \hline \\ does \quad not \quad necessarily \end{array}$ MP ≤<sub>m</sub> Empty mean Bisre  $\overline{MP} \leq_m \mathcal{E}mpty \longrightarrow \mathcal{E}mpty$  is not re not re (``if MP re <u>MP</u> re NP recursive)

Takeaway: 1) To show B is undecidable, reduce a known undecidable problem A to B. 2) To show B is not r.e. Give a many-one reduction from a known not r.e. set A Suppose now you want to show that a language is not  $\gamma \cdot e$ .  $\overline{MP} \leq_m L$   $\overline{MP} \leq_m L$ 





 $\langle M \rangle \xrightarrow{h} \langle M', Mau \rangle$  accepts every string same  $\Xi^*$   $L(Mau) = \Xi^*$  $L(M) = \phi \implies L(M') = \phi$  $L(M) \neq \phi \implies L(M') = \leq^*$ Not possible How to find M'? <u>Claim 3</u>: Empty ≤m EQ ⇔ Empty ≤m EQ

M': on input y 1) Simulate M on every input with larger and larger time

2) If M accepts some string, then M' accepts y.

If  $L(M) \neq \phi$ , then  $L(M') = \leq^*$ If  $L(M) = \phi$ , then  $L(M') = \phi$ 

 $\therefore$  Both EQ and  $\overline{EQ} \longrightarrow not$  re





If Apr 2025
 Time = # of steps

 Time = # of steps
 Space = # cells accessed (+ input space)

 FA
 
$$a \not a \not a \not b \not b \not b \dots$$

 FA
  $a \not a \not a \not b \not b \end{pmatrix}$ 

 A = { $a^n b^n \mid n > 0$ }

 1) Scan the input to check  $a^* b^*$ 

 2) Scan back and forth to cross off one a and b in each iteration.



## Example at first : A $\in DTIME(n^2)$ $\downarrow D = deterministic$

DSPACE (S(n)) := { B | B recognized by M using space S(n)}
No restriction on time no requirement of halting.
One tape turing machine: S(n) > n
Other models: S(n) → Space used on the work tape.
Homework: you can always build an equivalent halting TM



## Connected := { <G> | G is undirected, connected graph }

- NDTM M Guess this spanning tree

n<sup>2</sup> edges vrite / 2 logn bits per edge 3 \* Guess a spanning tree O(n logn) bits Non-determinism \* Verify if it is indeed  $O(n^2)$ You are not You are not exploring all branches at **\*** Accept iff verification passes once. NTIME  $(n^2)$ 

Ν



