03 Mar 2025 - Theory of Computation

→ 1.41, 1.42

66 in Sipser

$$M = (Q, \Sigma, \delta, q_0, F) \text{ DFA}$$

One of the states is called "home" state.

Let $h \in Q$ be the home state.

A string $s \in \Sigma^*$ is called synchronizing sequence if $\forall q \in Q$.

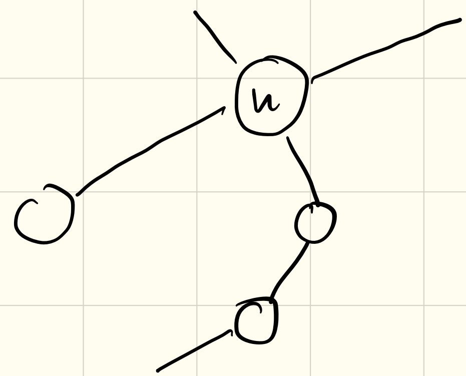
$$\hat{\delta}(q, s) = h$$

Call M synchronizable if $\exists$ a synchronizing sequence for some state $h \in Q$.

Prove : If M is a k-state synchronizable DFA then it has a synchronizing seq. of length $\leq k^3$.

If s is a synchronising sequence wrt h, then $\forall\, \alpha \in \Sigma^*$, αs is also a synchronizing sequence wrt h.

$$\begin{pmatrix} q_0 \\ \vdots \\ q_{k-1} \end{pmatrix} \xrightarrow[\leq k^k]{s} \begin{pmatrix} h \\ h \\ \vdots \\ h \end{pmatrix}$$



1.46

Use closure properties

$$\{0^m 1^n \mid m \neq n\}$$

Suppose regular :

1. Complement
2. Intersection with $0^* 1^*$
3. result $=$ $0^n 1^n$ $\rightarrow$ not regular

$\Rightarrow\Leftarrow$

$\boxed{1.47}$, 1.56

11 , 101 , 1001 , 10001 , 100001

10 , 12 , 21 , 122 , 1002

$7 = 3 \times 2 + 1$

$33 = 3 + 9 +$

$3^3 + 3 \times 2$

$17 = 3 \times 5 + 2$

$= 3 + 3^2 + 3 + 2$

$= 3 \times 2 + 3^2 + 2$

Let A be any regular language. Show that the set $\{ x \mid \exists y \ |y| = |x|^2 \text{ and } xy \in A \}$ is regular.

$$|y| = 2^{|x|} \longrightarrow \text{easier}$$

$$|y| = 2|x|$$