10 Feb 2025 – Theory of Computation – Week 06

$L = \{ x \in \{0, 1\}^* \mid \nexists$ a pair of 1s separated by odd # of symbols $\}$

↓

?? find minimal automata

$$(x, y) \in R_L \iff \forall z \in \Sigma^* \left(xz \in L \iff yz \in L \right)$$

→ algorithm to minimize automata

$$M = (Q, \Sigma, \delta, s, F)$$

$$p \approx q \iff \forall x \in \Sigma^* \left(\hat{\delta}(p, x) \in F \iff \hat{\delta}(q, x) \in F \right)$$

$$p \not\equiv q \quad \text{if} \quad \begin{array}{l} p \in F \\ q \in Q \setminus F \end{array}$$

Context - Free Grammars

used in parsers

$$G_1 \left[\begin{array}{lcl} S & \rightarrow & 0S1 \\ S & \rightarrow & R \\ R & \rightarrow & \varepsilon \end{array} \right]$$

Variables: list of symbols appearing on the left side of a rule.

Terminals: list of symbols other than variables

$$\left. \begin{array}{l} \text{vars} = \{S, R\} \\ \text{terminals} = \{0, 1\} \\ \text{start} = S \end{array} \right\} \text{finite}$$

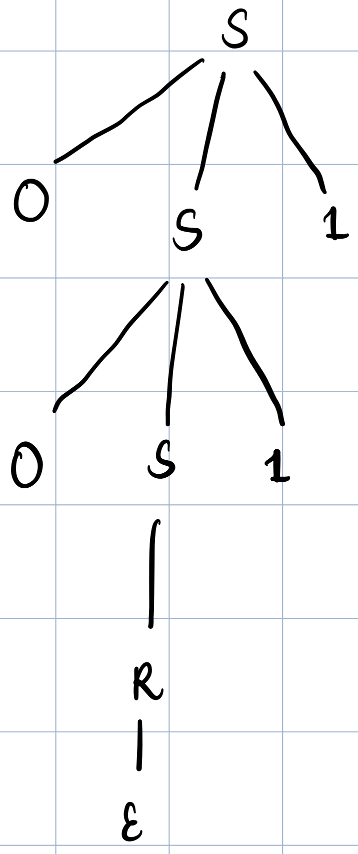
rules: $\text{vars} \longrightarrow (\text{var} \cup \text{ter})^*$

vars ⟶ a string over vars and terminals

start variable: one of the variable is designated as a start variable.

There is always a start variable

Tree of substitution



String generated

S

0 S 1

0 0 S 1 1

00 R 1 1

00 11 } end with a string that does not have variables

Repeat variable substitution until there are no variables in the string being derived.

At this moment, you have a string over terminals.

This string is said to be generated by grammar

$L(G)$:= set of all strings over terminals generated by G.

Regular languages	FA	Regular expression
Context-free languages	Push down automata	Context-free grammar

$L(G_1) = \{0^k 1^k \mid k \geq 0\}$ ⇝ prove by induction

A CFG is (V, Σ, R, s)

V : finite set of variables

Σ : finite set of terminals

R : finite set of rules

$s \in V$ is the start variables

$L \subseteq \Sigma^*$ is a CFL if $\exists$ a CFG G s.t. $L(G) = L$

terminal → . . .
is not
a valid rule.

ε is not a
part of the
terminal
but it can be
used

$1 S 0$ → not
allowed as start.
You can quickly
add a rule
to get here

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$

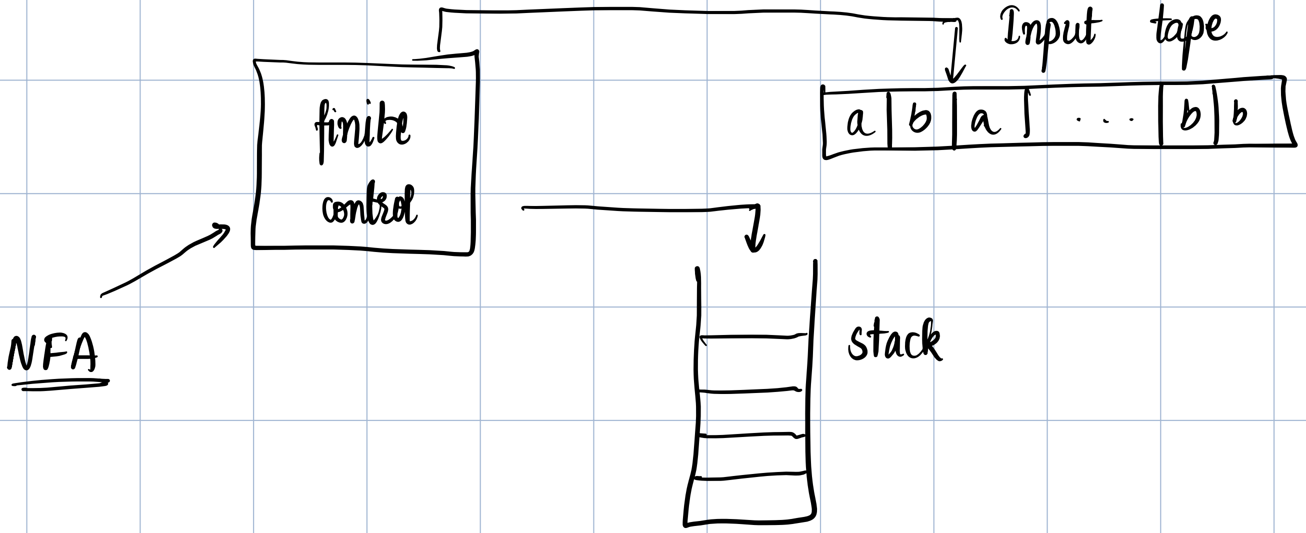
generates

palindrome $= \{x \in \{0,1\}^* \mid x = \text{reverse}(x)\}$ → cannot be done with automata

Does every regular language have a CFG ?

Push down automata (PDA)

Public Display of Affection



Properties

1) Input tape is read-only. Starts at the leftmost alphabet and moves right only.

Configuration of a machine → at a point of time

$$\left(\underset{\substack{\uparrow \\ \text{state}}}{q},\ \underset{\substack{\uparrow \\ \text{input} \\ \text{alphabet} \\ \text{head}}}{a},\ \underset{\substack{\uparrow \\ \text{stack alphabet}}}{b}\right)$$

$$(q, a, b) \longrightarrow \left(q', \underset{\substack{\downarrow \\ \text{stack}}}{c}\right)$$

Note : I did not attend the classes on 12 Feb and 13 Feb.