

$$het \quad \mathfrak{S} = \left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{12}} \right), \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \right\} \quad \|(1, 1)\| = \sqrt{2}.$$

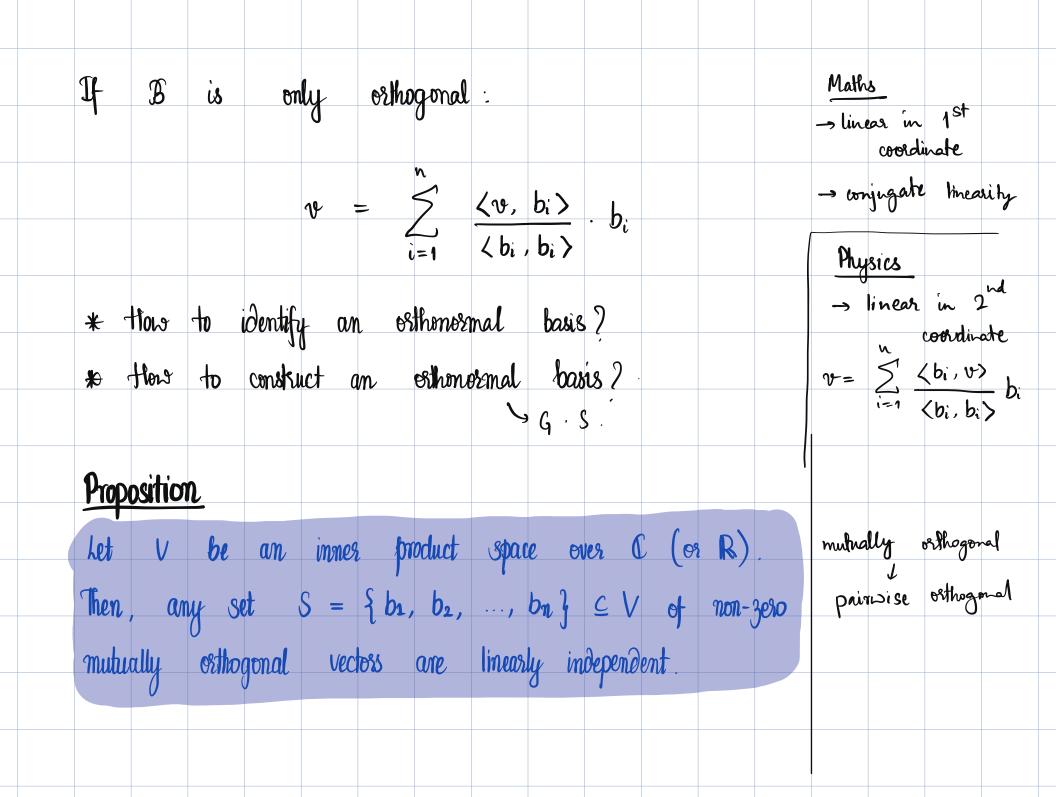
$$(1, 1) = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right)$$

$$(1, 1)\| = \left($$

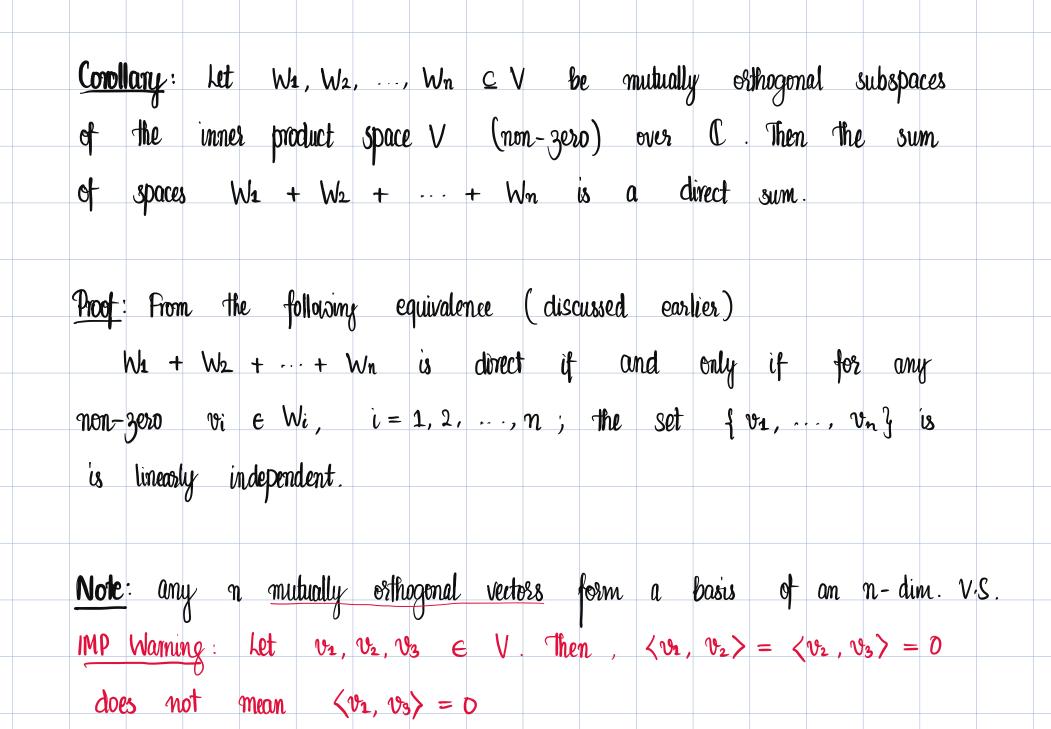
Let V be an inner product space, then any two subspaces W_1 , $W_2 \subseteq V$ are said to be mutually esthogonal if for any $x \in W_1$ and $y \in W_2$, $\langle x, y \rangle = 0$

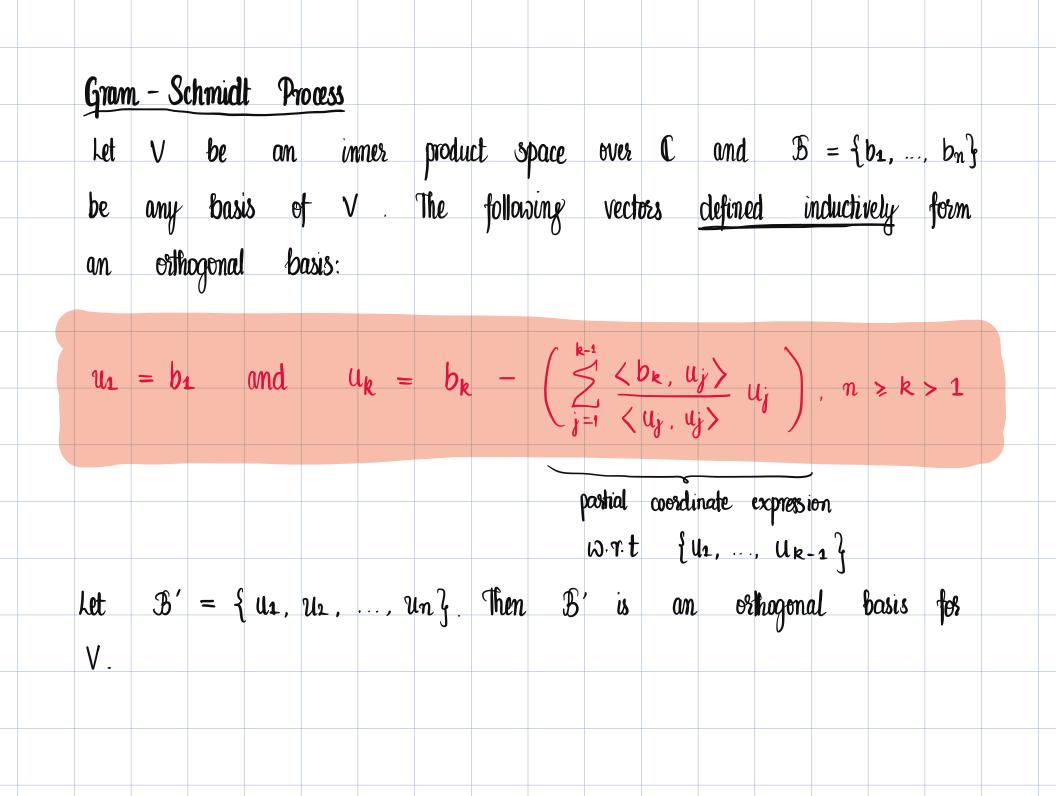
ObservationInaninnerproductspaceknowinganerthonesmalbasishelpsineasyidentificationofcoordinates.hetVbeaninnerproductspaceover0and $\mathcal{B} = \{b_1, \ldots, b_n\}$ beanesthonosmalbasisforVhetvbeanordinaryvectorinbeanesthonosmalbasisforVhetvbeanordinaryvectorinVSinceBisabasisthereexistuniquevr.vz.v.eCsuchthat $v = \tilde{Z}, v_i b_i$ iiiiiiiii

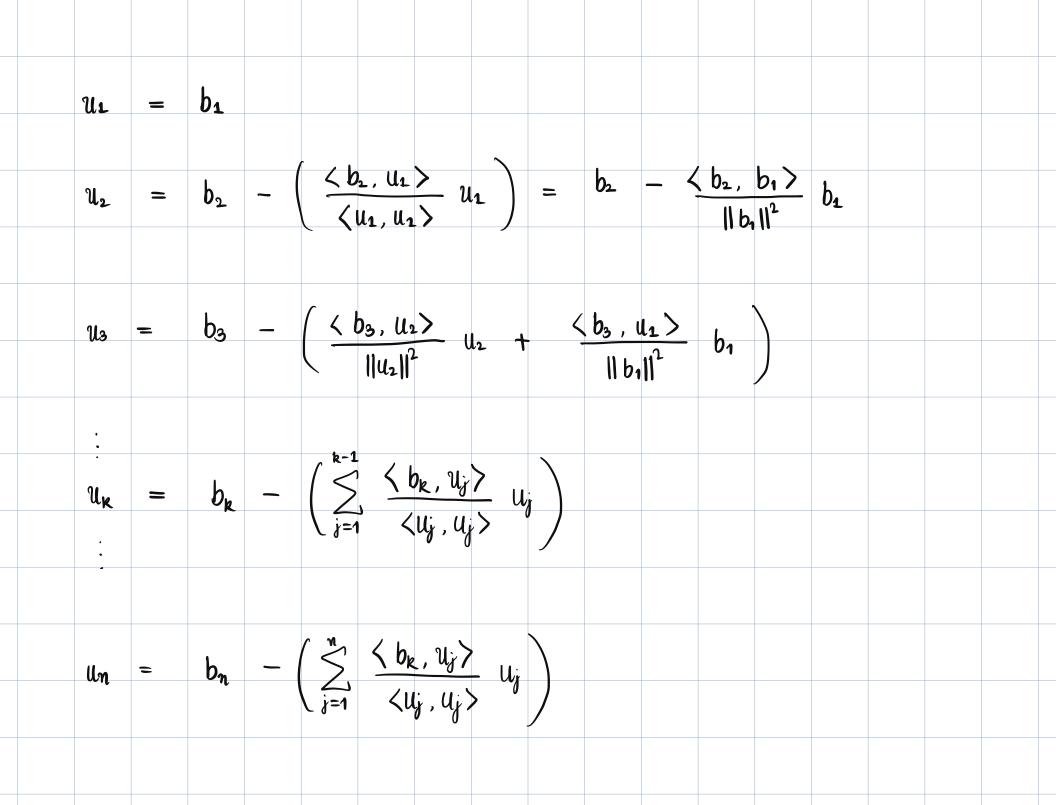
But since
$$\mathcal{B}$$
 is esthometrical, $\langle b_i, b_j \rangle = \delta_{ij}$ for $j = 1, ..., n$
 $\langle v, b_j \rangle = \langle \sum_{i=1}^{n} v_i b_i, b_j \rangle$
 $= \sum_{i=1}^{n} v_i \langle b_i, b_j \rangle$
 $= v_j$
So, the coexclinate representation of any $v \in V$ with respect to an
esthometrical basis $\mathcal{B} = \{b_1, ..., b_n\}$ is given by
 $v = (\langle v, b_1 \rangle, \langle v, b_2 \rangle, ..., \langle v, b_n \rangle)$



Proof: let
$$\mathfrak{F} = \{b_{1}, b_{2}, ..., b_{n}\}$$
 be anutually $e_{\mathfrak{F}}: dimensions 5$
 $f_{\mathfrak{H}}: b_{\mathfrak{h}}: \dots b_{\mathfrak{h}}\}$
orthogonal non-gove vectors.
 $het \alpha_{\mathfrak{h}}, ..., \alpha_{\mathfrak{h}} \in \mathbb{C}$ such that $\sum_{i=1}^{\mathfrak{h}} \alpha_{i} b_{i} = \mathbb{D}$
 $f_{\mathfrak{K}}: j$ in between $1, ..., \mathfrak{n}$. Then
 $\left\langle \sum_{i=1}^{\mathfrak{h}} \alpha_{i} b_{i}, b_{j} \right\rangle = \mathbb{O}$
 $\left\{ \begin{array}{c} \cdots \langle 0, v \rangle = \langle v, 0 \rangle \\ = \mathbb{O} \end{array} \right.$
 $va_{\mathfrak{h}}: \dots, va_{\mathfrak{h}} \in W_{\mathfrak{h}}, \dots, va_{\mathfrak{h}} \\ va_{\mathfrak{h}}: \dots, va_{\mathfrak{h}} \in W_{\mathfrak{h}}, \dots, va_{\mathfrak{h}} \\ \Rightarrow \alpha_{\mathfrak{h}} \langle b_{\mathfrak{h}}, b_{\mathfrak{h}} \rangle = \mathbb{O}$
 $\left\{ \begin{array}{c} \cdots \langle 0, v \rangle = \langle v, 0 \rangle \\ = \mathbb{O} \end{array} \right.$
 $va_{\mathfrak{h}}: \dots, va_{\mathfrak{h}} \in W_{\mathfrak{h}}, \dots, va_{\mathfrak{h}} \\ \Rightarrow sum is direct \\ \Rightarrow \alpha_{\mathfrak{h}} \langle b_{\mathfrak{h}}, b_{\mathfrak{h}} \rangle = \mathbb{O}$
 $sum is direct \\ \Rightarrow \alpha_{\mathfrak{h}} \langle b_{\mathfrak{h}}, b_{\mathfrak{h}} \rangle = \mathbb{O}$
 $sum is direct \\ \Rightarrow \alpha_{\mathfrak{h}} = \mathbb{O}$
 $d_{\mathfrak{h}}: \langle b_{\mathfrak{h}}, b_{\mathfrak{h}} \rangle \neq \mathbb{O}$
 $sum is direct \\ \Rightarrow \alpha_{\mathfrak{h}} = \mathbb{O}$
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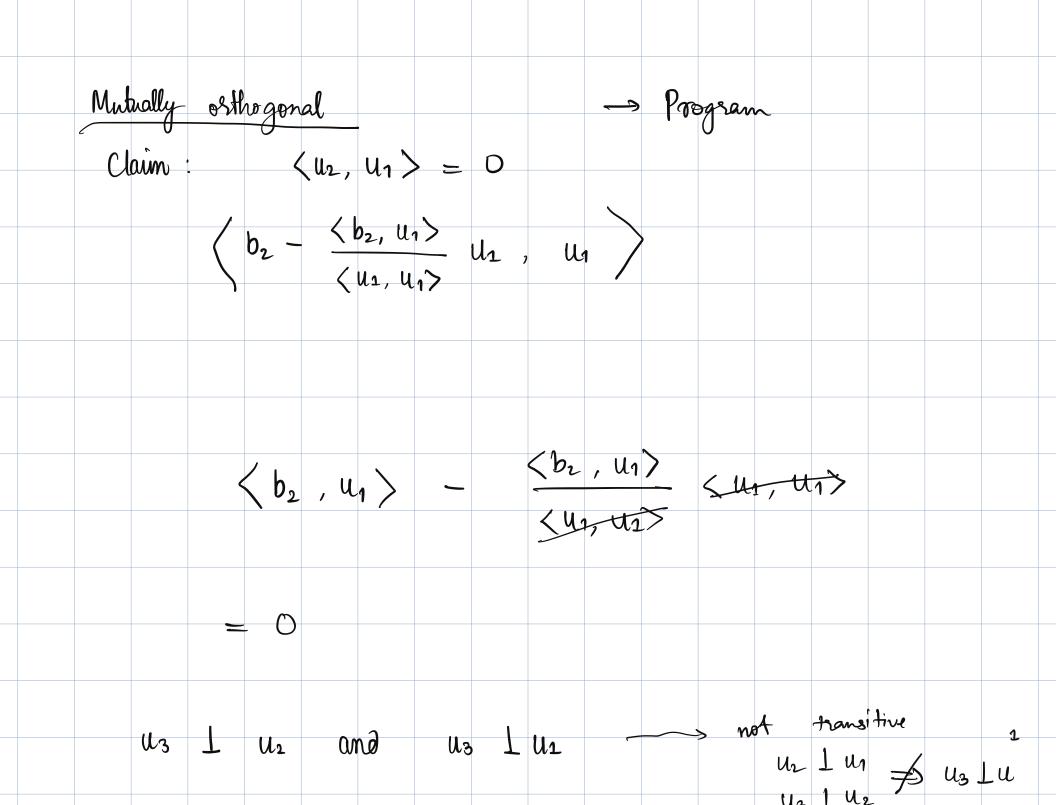


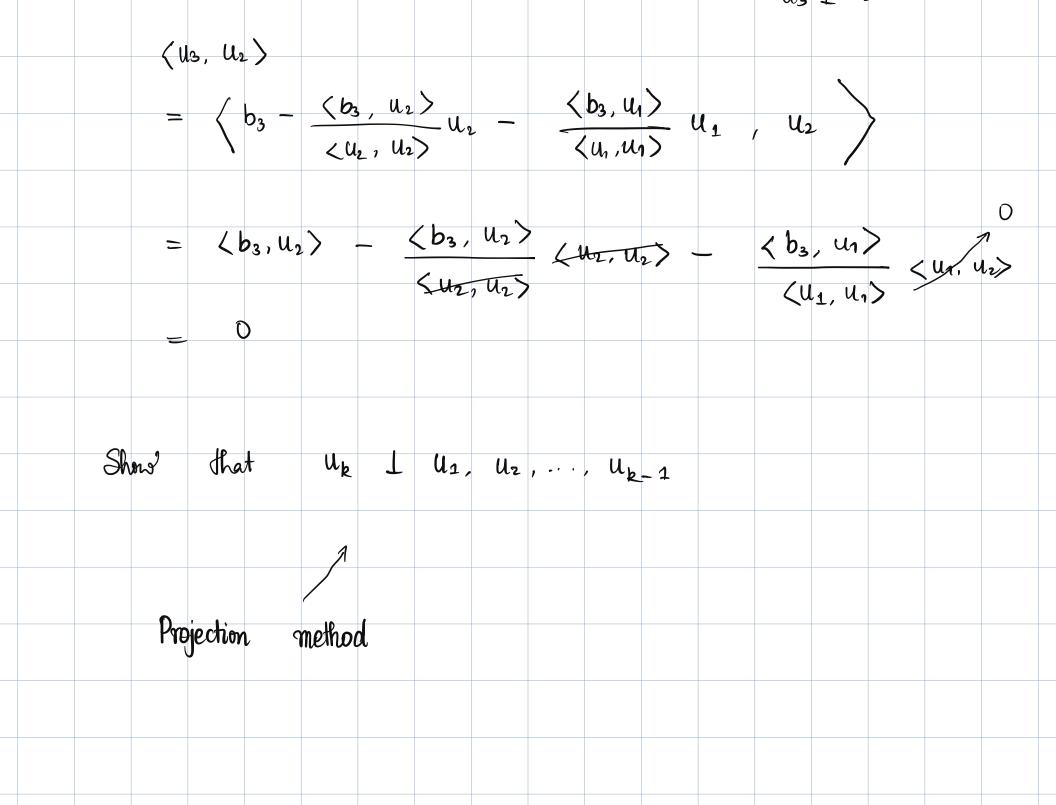
$$\begin{array}{c} \underline{\text{Claim:}} & \{u_{4}, \dots, u_{n}\} \rightarrow an \text{ orthogonal basis} \\ \hline \\ & \text{Show} \\ \hline \\ & \text{O} \quad u_{j} \neq 0 \quad \forall \quad j = 1, \dots, n \\ & u_{j} \quad u_{j} \quad cannot \quad be \quad 3^{e_{2}o} \\ \hline \\ & \text{O} \quad (u_{i}, u_{j} \rangle = 0 \quad if \quad i \neq j \\ & \text{basis element} \\ & u_{i} = b_{1} \quad basis \quad element \\ & u_{i} = b_{1} \quad bnon \quad 3^{e_{2}o} \\ & u_{2} = b_{2} - \left(\begin{array}{c} \langle b_{2}, u_{i} \rangle \\ \langle b_{2}, u_{i} \rangle \\ \langle b_{2}, u_{i} \rangle \end{array} \right) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid b_{i} \mid b_{i} \mid b_{i}) \\ \hline \\ & \text{O} \quad (u_{i} \mid b_{i} \mid$$

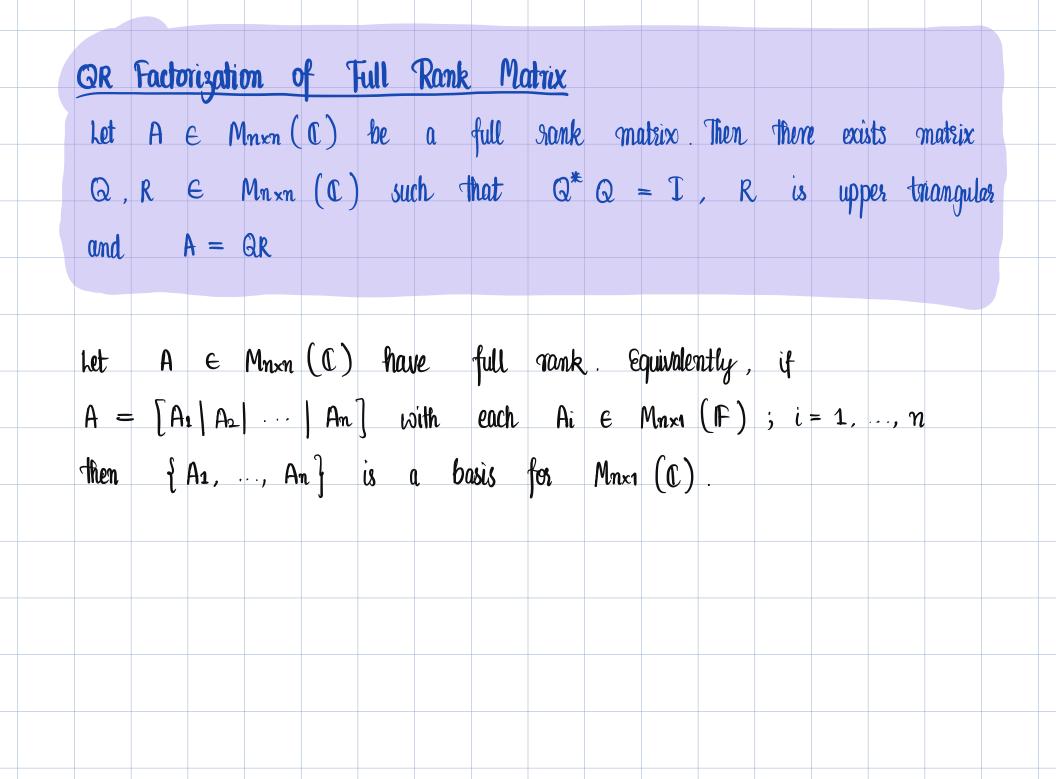
Suppose
$$u_{1} = 0$$

 $b_{2} = \frac{\langle b_{1}, u_{1} \rangle}{\langle b_{2}, u_{1} \rangle}$
 b_{4}
 $\Rightarrow \leftarrow$
Babis LT
 $u_{1} \neq 0$
 $u_{1} \neq 0$
 $u_{2} \neq 0$
 $u_{3} = 0$
 $b_{3} = \frac{\langle b_{3}, u_{1} \rangle}{\langle b_{3}, u_{2} \rangle}$
 $b_{2} + \frac{\langle b_{3}, u_{4} \rangle}{\langle b_{3}, u_{2} \rangle}$
 u_{2}

 $span \left\{ u_{1}, u_{2} \right\} = span \left\{ b_{1}, b_{2} \right\}$ Shows that $y_j \neq 0$ for any j = 1, 2, ..., n







Using Gram Schmidt:
$$U_{k} = A_{2}$$

 $U_{k} = A_{R} - \left(\sum_{j=1}^{k} \frac{\langle A_{k}, U_{j} \rangle}{\langle U_{j}, U_{j} \rangle} U_{j}\right), \quad k = 2, ..., n$
is an esthogonal basis
Equivolently, $A_{2} = U_{2}$
 $A_{R} = U_{R} + \sum_{j=1}^{k-1} \frac{U_{j}^{*}}{U_{j}^{*}} U_{j}$
Then, for $U = [U_{2} \mid U_{2} \mid ... \mid U_{n}]_{n \times n}$ and $G = \begin{bmatrix} 1 & \frac{U_{k}^{*} A_{k}}{U_{k}} & \frac{U_{k}}{U_{k}} & \frac{U_{$

Furth	vr ,	define		Qr	=(Ur In	·) ^{1/2}	k	= 1		, h				
		Q1 ,													
Then.	, th	e M	atrix	Q	:=	QL	Qı) Qn] e	Mns	an (O)		
	and		G ≭	Q =				[Q1	Q 2		. Q	~]			
						Q.									
						T									

