03 Feb 2025 - Intro to Statistics

→ Test on mean : two sample tests

Before

$$X_{11}, \; \ldots, \; X_{N_1 1} \sim N(\mu_1, \sigma_1^2)$$

$$x_{21}, \; \ldots, \; x_{n_1 1} \sim N(\mu_1, \sigma_1^2)$$

After

$$X_1, \; \ldots, \; X_N \sim N(\mu_2, \sigma_2^2)$$

$$x_{12}, \; \ldots, \; x_{n_2 2} \sim N(\mu_2, \sigma_2^2)$$

Recap:

One sample test for mean

- $pop^n \sim$ normal
  - σ known
    - z test
  - σ unknown
    - small n → t test
    - large n → z test or t-test
- $pop^n \sim$ non-normal
  - small n → non-parametric
  - large n → z-test

Two sample tests for mean

pop^n ~ normal

σ known ⟿ both σ_1 and σ_2 are known

z-test

$$\frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

σ unknown

t- test

$\sigma_1^2 \neq \sigma_2^2$

$$\frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\left.\sigma_1^2 = \sigma_2^2\right\}$ F-test

$$\frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s_1^2 = \left(\frac{1}{n_1 - 1}\right)\sum_i (x_{1i} - \bar{x}_1)^2$$

$$s_2^2 = \left(\frac{1}{n_2 - 1}\right)\sum_i (x_{2i} - \bar{x}_2)^2$$

If both σ_1^2 and σ_2^2 are same,

we can use weighted sample variance.

$$\underbrace{S_p^2}_{\text{pooled average of sample variance}} = \left.\frac{(n_1 - 1)\,S_1^2 + (n_2 - 1)\,S_2^2}{(n_1 - 1) + (n_2 - 1)}\right\} \text{pooled estimator}$$

$$X \qquad x_{11}, \ \ldots, \ x_{1n} \sim N(\mu_1, \sigma^2)$$

$$\qquad x_{21}, \ \ldots, \ x_{2n} \sim N(\mu_2, \sigma^2)$$

How do we know if population variances are different?

F - test for variances

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_a : \sigma_1^2 \neq \sigma_2^2$

using spreadsheet

F. TEST

$F_{\text{test stat}} = \dfrac{S_1^2}{S_2^2}$; $F_{\text{critical}} = F_{(1-\alpha);\, \underbrace{n_1, n_2}_{\text{two degrees of freedom}}}$

$F_{obs} > F_{crit} \implies$ Reject H_0

larger s^2 goes into the numerator

Example: see google sheets and pdf

μ_1, σ_1^2 : before

μ_2, σ_2^2 : after

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_2 > \mu_1$

$\mu_1 < \mu_2$

Why?

- → protecting old ideas
- → We don't want more serious error
- → new / risky is H_a

$H_0 : \underbrace{\mu_1 - \mu_2 = 0}$ $\qquad \alpha = 0.01$

$\downarrow$

μ_0 null value

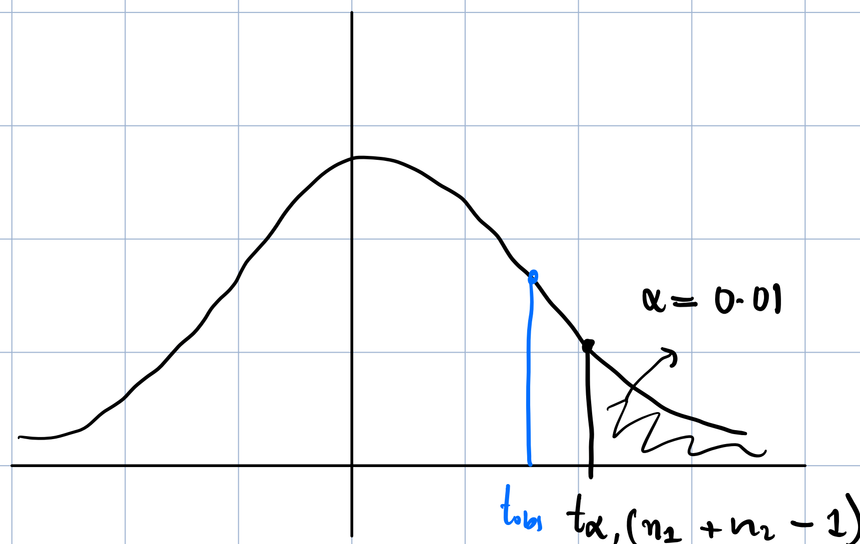
F.TEST(A2 : A10, B2 : B10) $\approx 0.93 \longrightarrow$ p-value

$F_{obs} = 1.034$

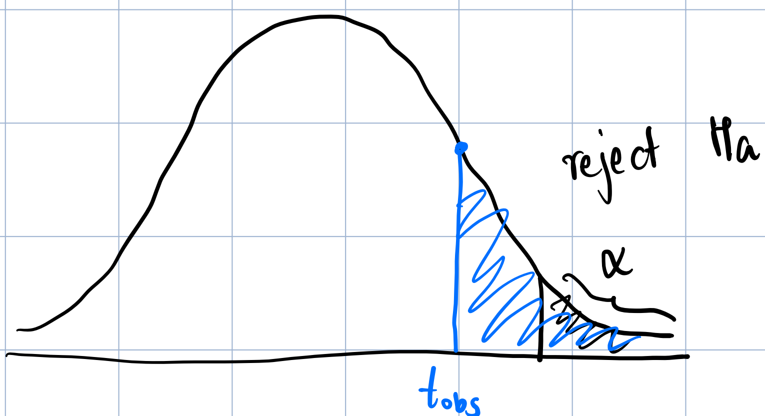
$$\text{p-value} > \alpha \longrightarrow \text{reject } H_0$$

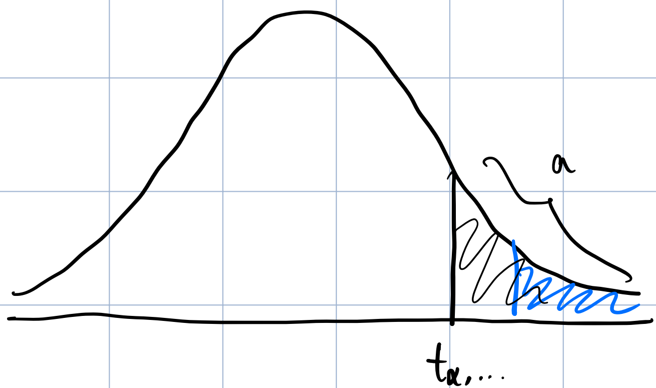
$$t_{obs} = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= -3.4$$



Right tailed test





Reject H_0

Probability to the right of t_{obs}

$= p$ - value

$= P(t > t_{obs})$

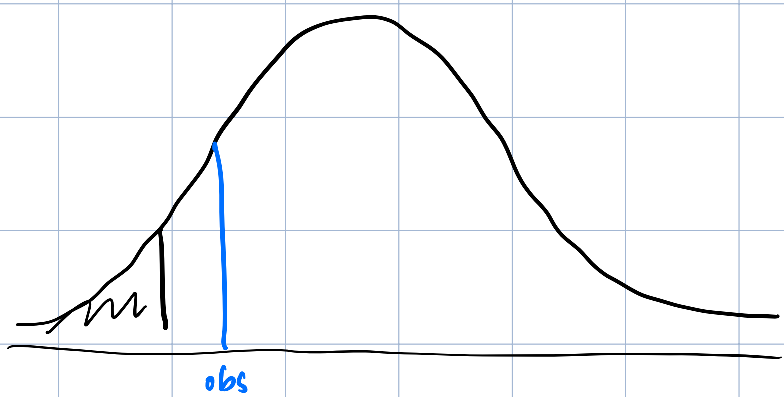
$$\boxed{\text{p-value} < \alpha \Rightarrow \text{reject } H_0}$$

For a left tailed test

pvalue = probability to the left of t_{obs}

$= P(t < t_{obs})$

$$\boxed{\text{pvalue} < \alpha \Rightarrow \text{reject } H_0}$$



5 Feb 2025

→ Two sample test for proportion won't be asked.

→ F-TEST results will be provided.

Problem: Employee suggestions . . .

→ First fix alternative hypothesis

"significant difference"

quantify it

$H_0 : \mu_1 = \mu_2$

$\mu_1 - \mu_2 = 0 = \mu_0$

$H_a : \mu_1 \neq \mu_2$

$\bar{x}_1 = 5.8 \qquad n_1 = 36 \qquad \sigma_1^2 = 1.7^2$

$\bar{x}_2 = 5.0 \qquad n_2 = 25 \qquad \sigma_2^2 = 1.4^2$

σ known $\longrightarrow$ z-test

We don't need F-test here

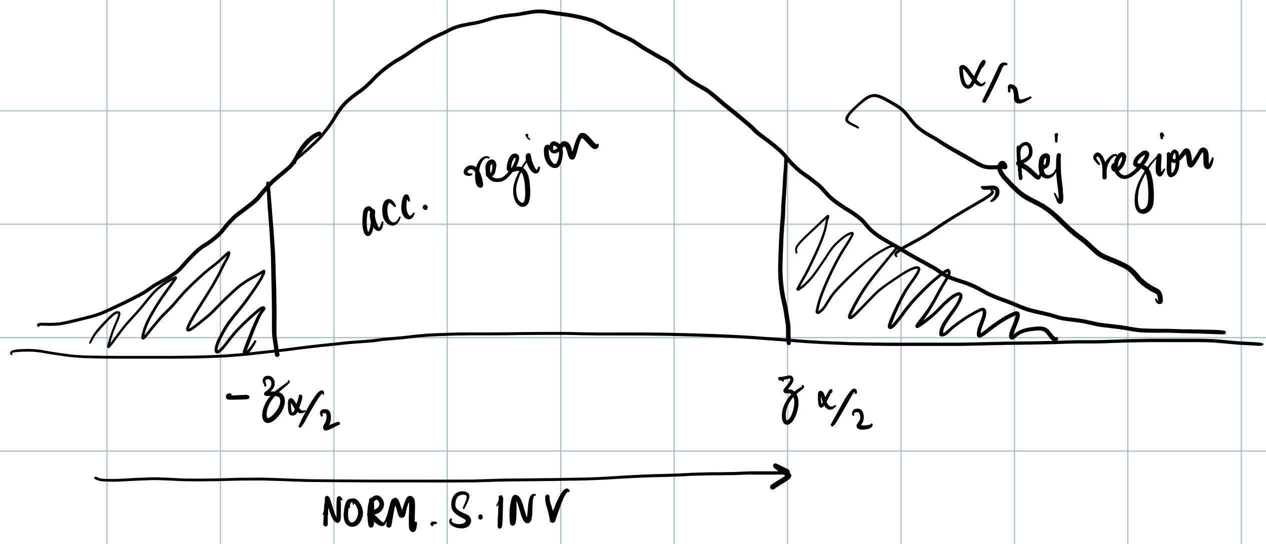
$$z_{\text{test stat}} = \frac{\bar{x}_1 - \bar{x}_2 - \mu_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z_{obs} = 2.008$$

Assumptions : ① independent ② normality

$X_{11}, \cdots, X_{n1} \sim N(\mu_1, \sigma_1^2)$

$X_{12}, \cdots, X_{n2} \sim N(\mu_2, \sigma_2^2)$



$$z_{\alpha/2} = \text{NORM.S.INV}(1 - 0.025)$$

takes probability to the left

$$|z_{obs}| > z_{\alpha/2} \Rightarrow \text{Reject } H_0$$

Critical value method

p-value method

right-tailed test : pvalue = area to the right of Z_{obs}

left-tailed test : p-value = area to the left of Z_{obs}

two-sided test : p-value = 2 × area to the right of Z_{obs}

p-value	critical value
our input: Value	our input: probability
spreadsheet: Probability	spreadsheet: Value
NORM.S.DIST	
T.DIST	

$$\text{NORM.S.DIST}(z_{obs}) = 0.977$$

$$1 - \text{NORM.S.DIST}(z_{obs}) = 1 - 0.977$$

$$\text{p-value} = 2 \times (1 - 0.977)$$

$$= 0.0446$$

$\boxed{\text{p-value} < \alpha}$ → Reject H_0

evidence in support

p-value → amount of confidence you have on null hypothesis

p-value ⇝ more used

⇝ more known concept and easier to interpret.

When p-value is very close to α → we don't make strong claims (change dataset, chances of p-value to go to the other side ↑)

→ get more evidence (larger sample size)

4.

$H_0 : \mu_A \leq \mu_B$ ✓ $\qquad H_a : \underbrace{\mu_A > \mu_B}_{\text{given}}$

$\mu_A = \mu_B$ ✓

$\alpha = 0.1 \qquad \bar{x}_1 = \qquad n_1 = \qquad s_1^2 =$

$\bar{x}_2 = \qquad n_2 = \qquad s_2^2 =$

→ t-test

Assumptions : ① independence / i.i.d
② normality

$$t_{test\ stat} = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{}$$

} depends on F - test

F - test

H_0 : pop^n variances are equal

H_a : pop^n var are unequal

$$F_{obs} = \frac{S_2^2}{S_1^2} = 1.4$$

$$F_{crit} = F_{\frac{\alpha}{2};\, n_1, n_2}$$

$$= F.\,INV\,(0.95,\, n_1,\, n_2)$$

$$= 1.67$$

if $F_{obs} > F_{crit} \Rightarrow$ reject H_0

$$S_p^2 = \frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{(n_1 - 1) + (n_2 - 1)} =$$

if $t_{obs} > t_{\alpha; n_1+n_2-1} \implies$ Reject H_0