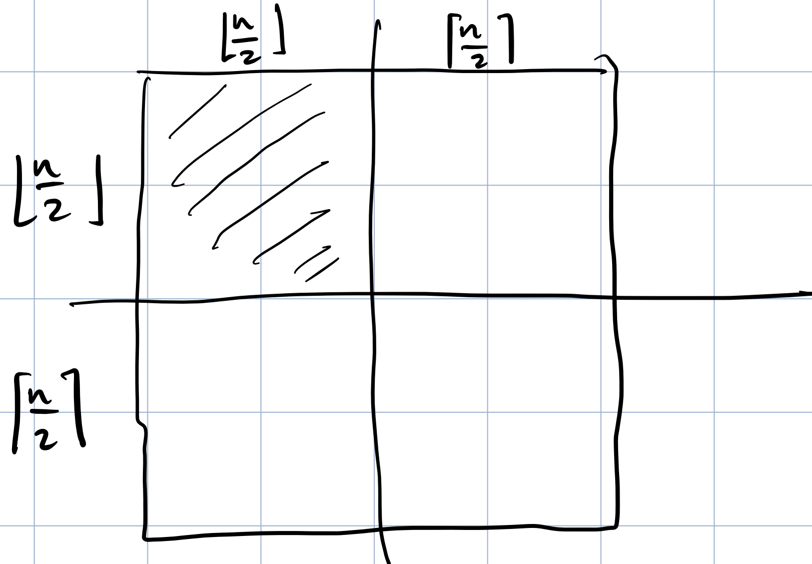
17 Feb 2025 – Algorithms – Week 08

Exam problems

3. Reduce to finding $\lfloor n/2 \rfloor^{th}$ smallest element

5. AAAH This was so easy ☹

6. Apply divide and conquer and reduce into solving in one of the 4 quads



$$T(n) = T\left(\frac{n}{2}\right) + O(n)$$

7. b. Radix sort

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2. $SCS(i, j)$ = length of shortest common supersequence of $A[1..i]$ & $B[1..j]$

$i = 1$

$SCS(1, j) = j$ if $A[1] \in \{B[1], \ldots, B[j]\}$

$= j + 1$ o/w

Recurrence

$A[1] \quad \ldots \quad A[i]$

$B[1] \quad \ldots \quad B[j]$

If $A[i] = B[j]$

$SCS(i, j) = SCS(i-1, j-1) + 1$

If $A[i] \neq B[j]$

$SCS(i, j) = 1 + \min(SCS(i-1), j) + SCS(i, j-1))$

4. $P(j, i) = Pr[\text{India wins atleast } j \text{ games in first } i]$
$0 \le j \le i$

$$Pr(j, i) = p_j \cdot pr(j-1, i-1) + pr(j, i-1) \cdot (1 - p_j)$$

$P(j, i) = Pr[\text{India wins exactly } j \text{ games in first } i]$
$0 \le j \le i$

Running time : $O(n^2)$

for $j = 0$ to n

  for $i = j$ to n

    find $P(i, j)$

Edit distance

Running time : $O(mn)$

Space : $O(mn)$

can improve to $O(m + n)$

How to backtrack?

??

Graph markup language

Dynamic Programming on Trees

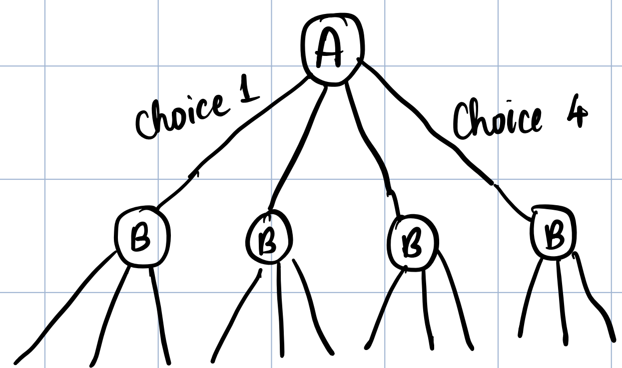
Input = tree, instead of a sequence

Game trees : 2 player games

First player
A

Second player
B

→ no chance involved in the outcome of each move.



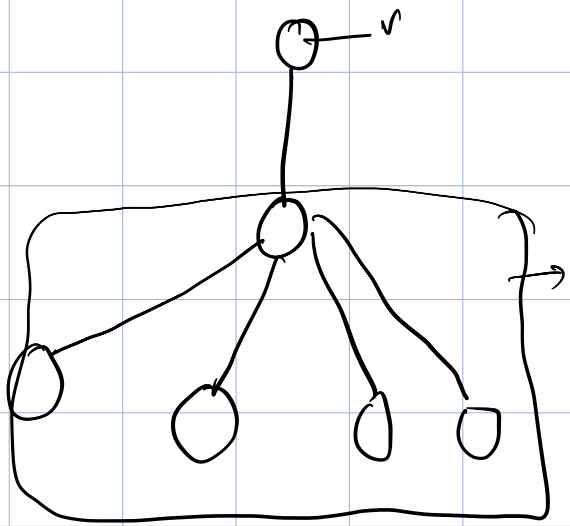
Leaf nodes represent end of game.
→ value represents who won the game.

Every finite 2-player game with no ties has a winning strategy for one of the players.

A is the first player → makes move first.

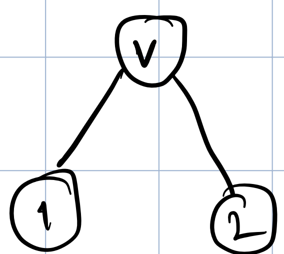
Possible Questions: Which player has a winning strategy and what is it?

Consider an arbitrary subtree

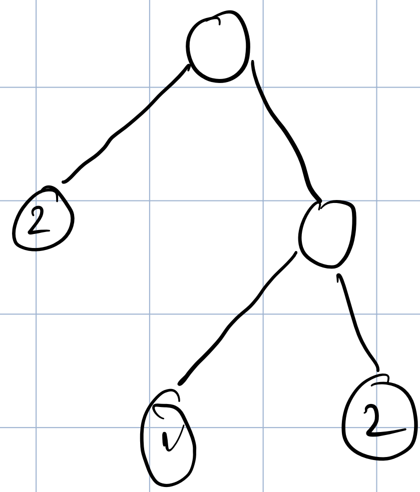


$f(v) = 1$ if player at v wins T_v

$= 0$ o/w

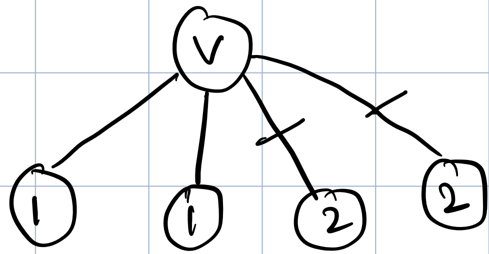


player 1 wins



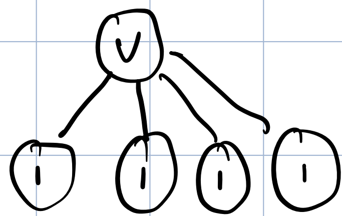
2 always wins

Compute $f(v)$ for v = leaves then parents of leaves and so on.



v can win

$f(v) = 1$



v cannot win

$f(v) = 1$

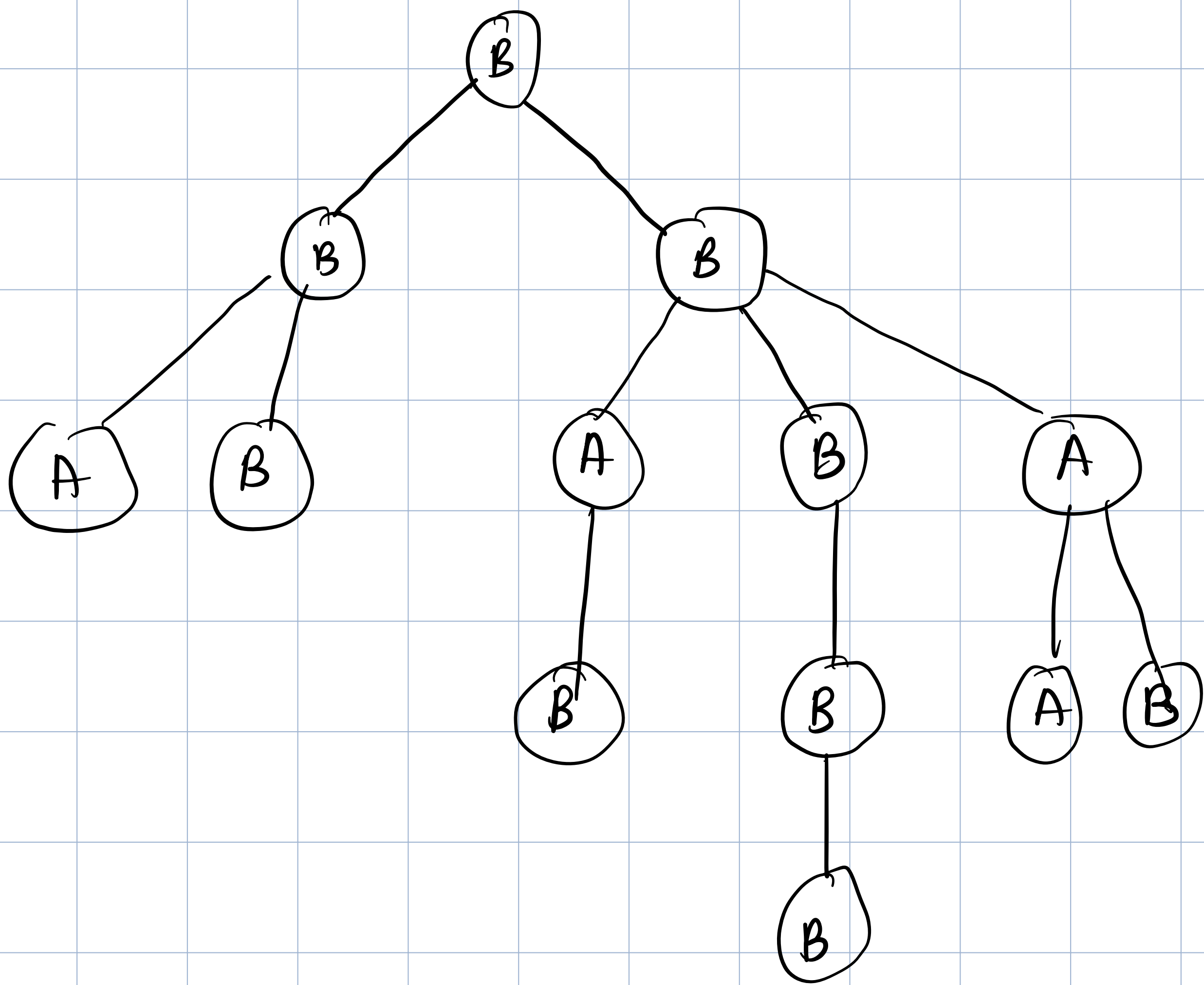
$f(v) = 2$ if

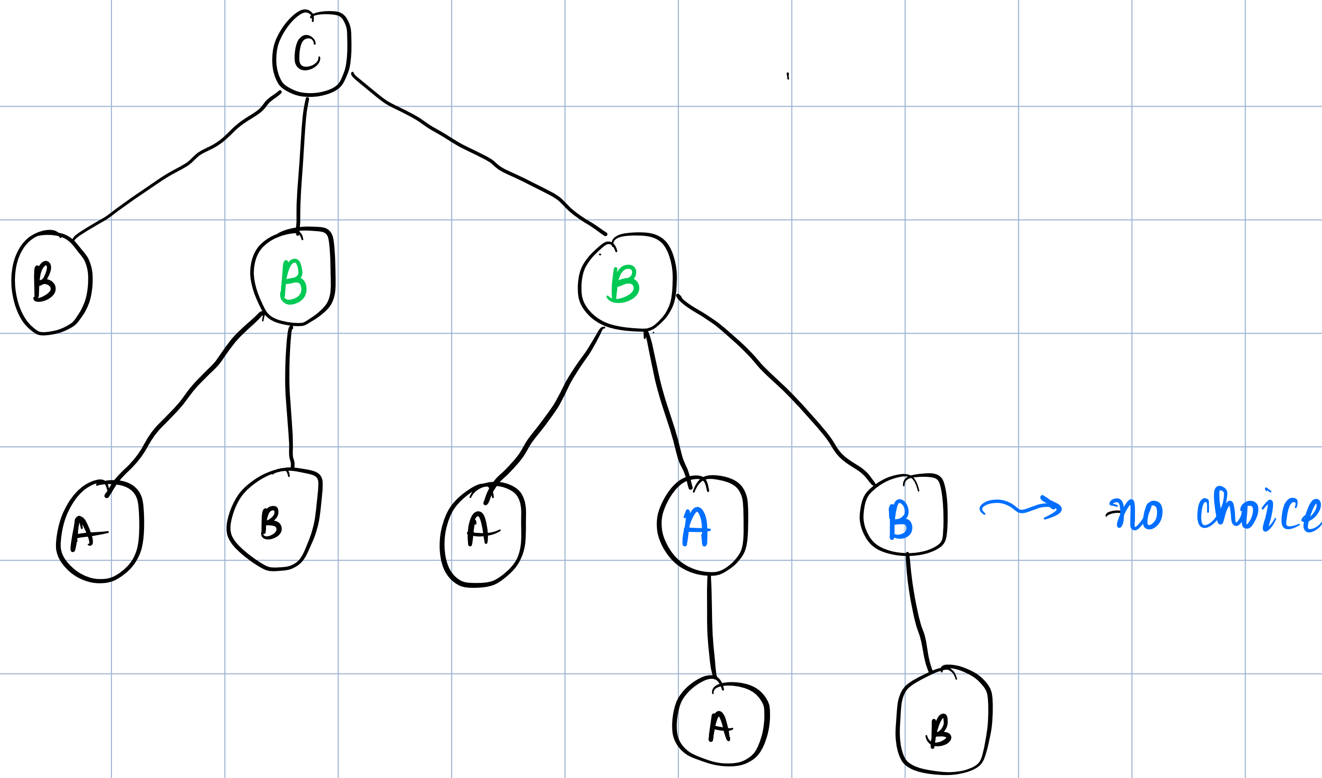
$f(w) = 1$ $\forall$ children w of v

A's turn at v :

$f(v) = A$ if at least one child w of B has $f(w) = A$

$f(v) = B$ if at least one child has $f(w) = B$


B
B
B
A
B
A
B
A
B
B
A
B
B


C
B
B
B
A
B
A
A
B
no choice
A
B