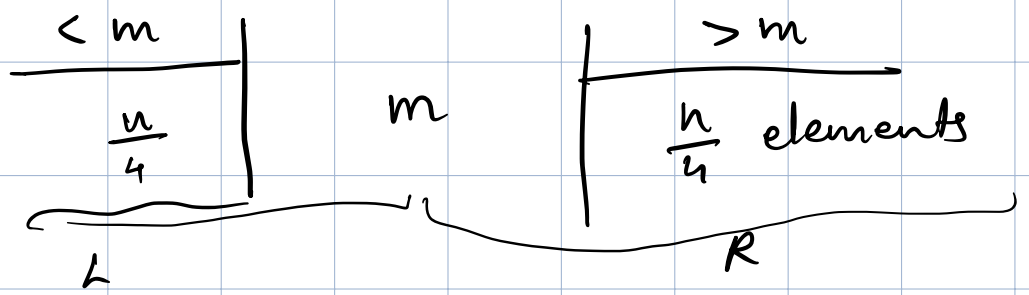
Algorithms - Week 04

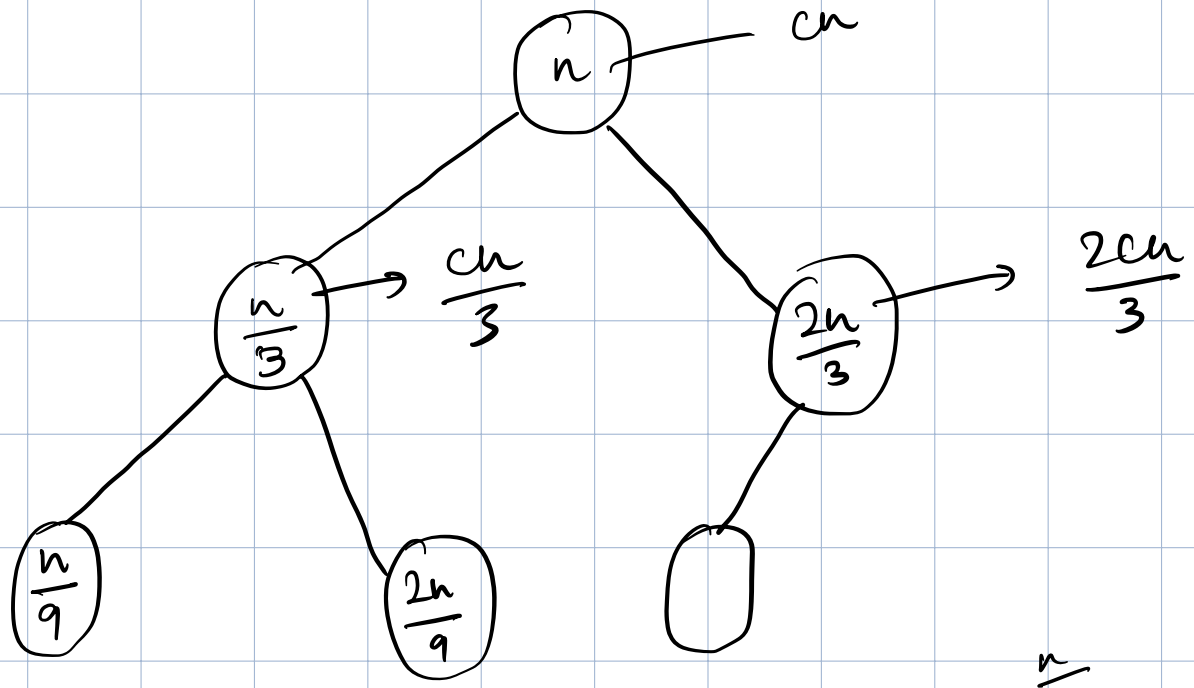
20 Jan 2024

$A[1, 2, \ldots, \frac{n}{2}]$ $\quad$ $A[\frac{n}{2}+1, \ldots, n]$

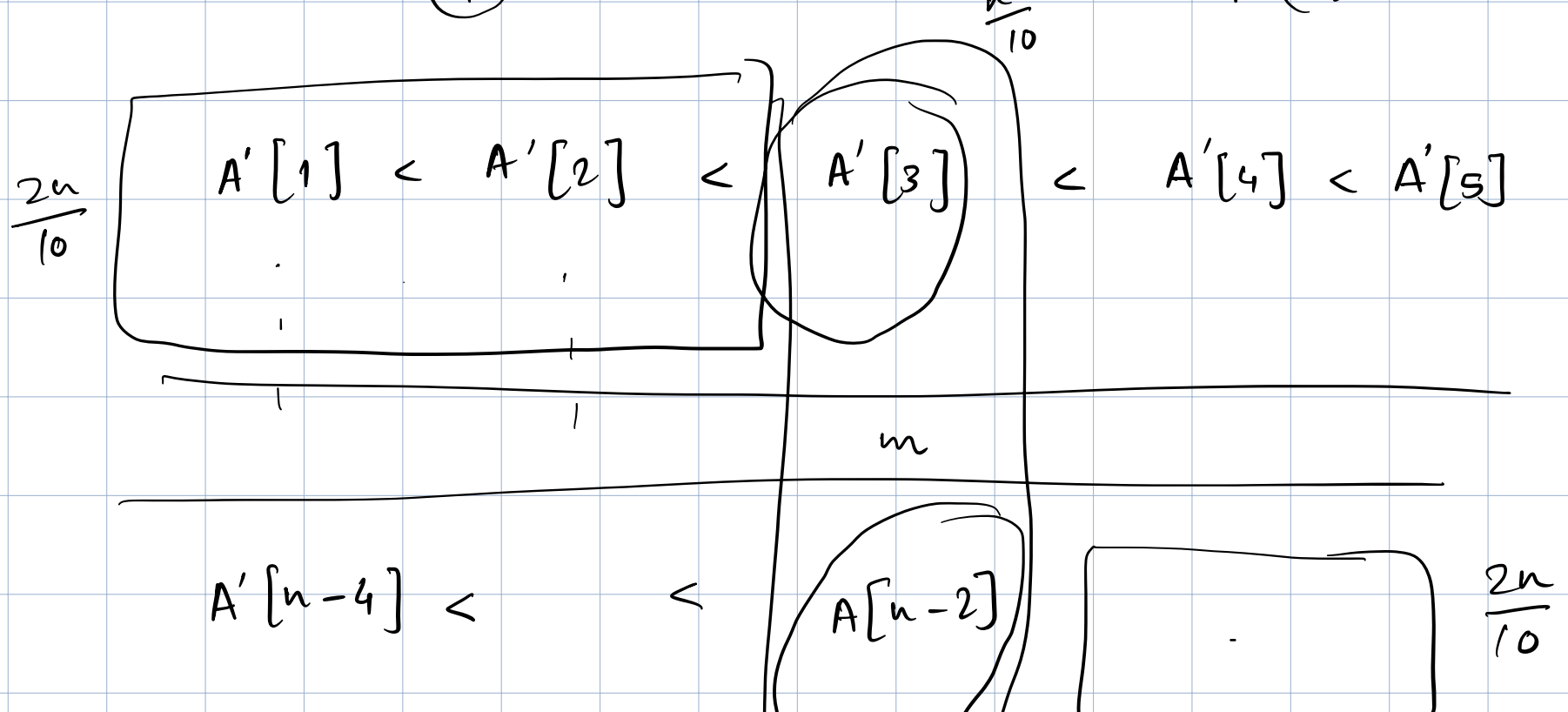
Median = m

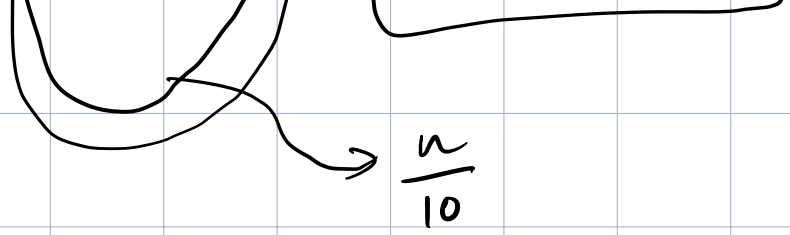


$T(an)$

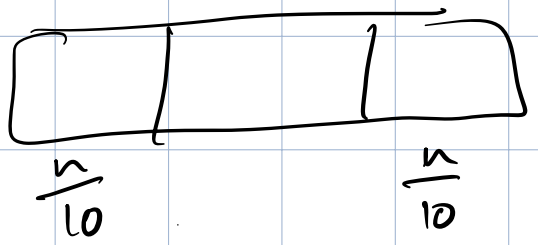


$T\left(\frac{n}{5}, 1\right)$

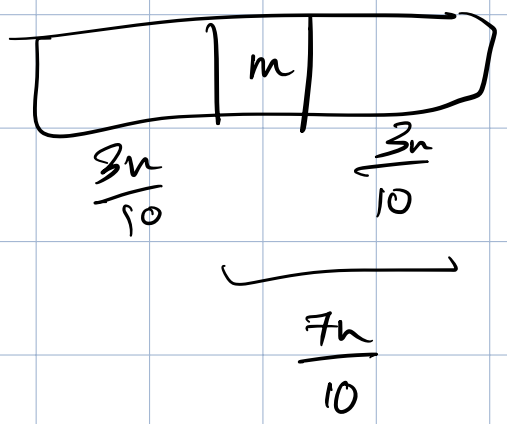




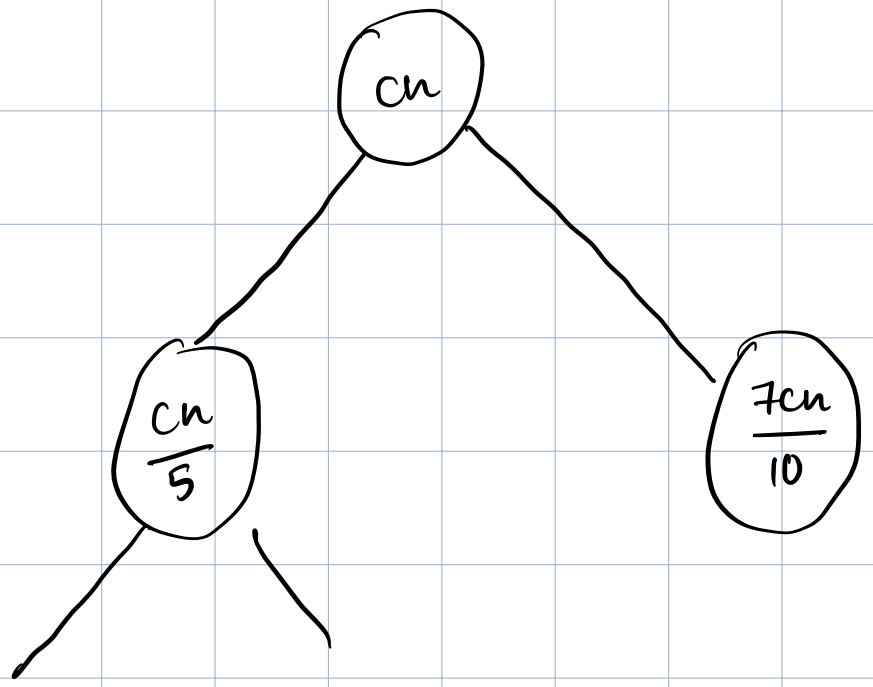
Before

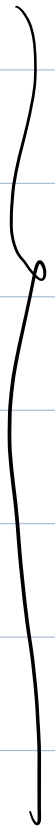


After



$$T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + O(n)$$





cn

$h = \log_5 n$

$\frac{9cn}{10}$

$\frac{81cn}{100}$

$$T(n) \leq cn + \frac{9cn}{100} + \frac{81cn}{100} + \cdots + cn\left(\frac{9}{10}\right)^{\log_{10/7} n}$$

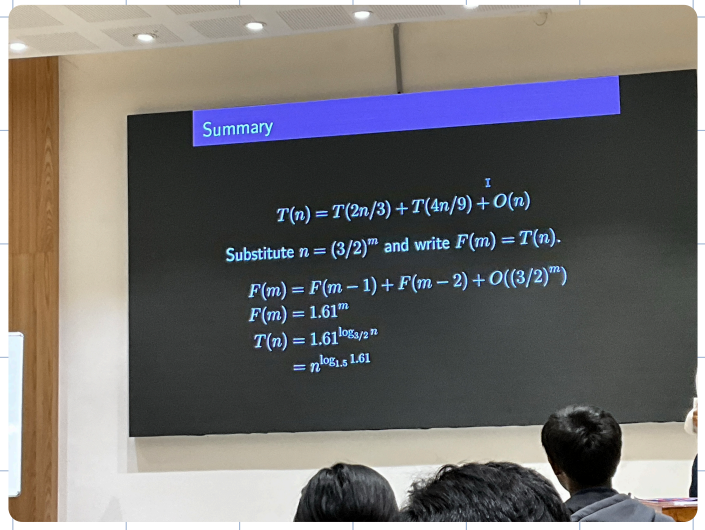
$a = cn$

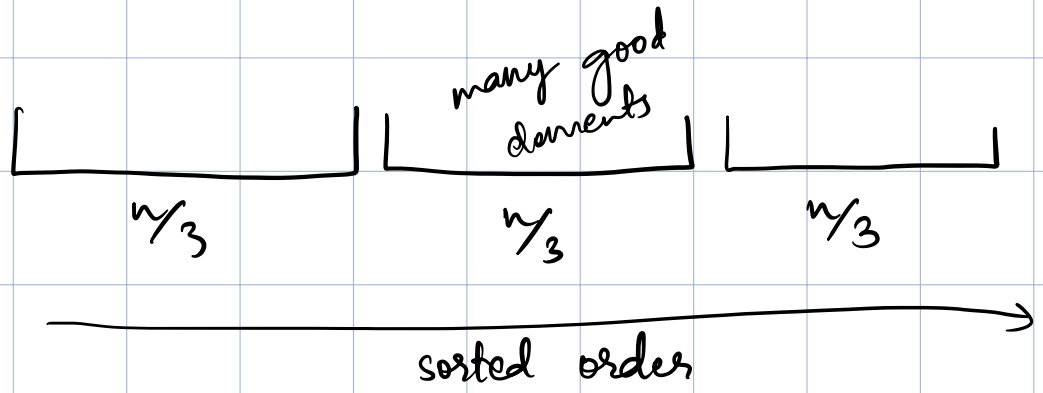
$r = 9/10$

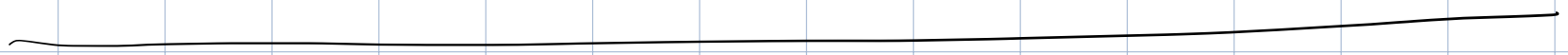
$h = \log_5 n$

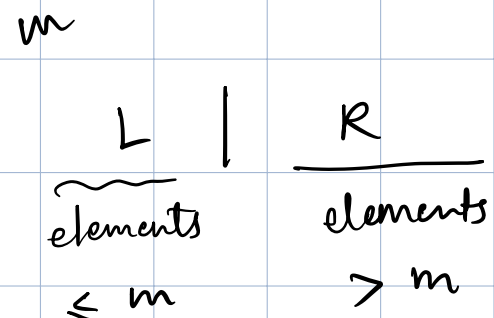
$$\leq \frac{cn\left(1-\left(\frac{9}{10}\right)^{\log_{10/7} n}\right)}{1/10}$$

$$T(n) \leq 10\left(cn - n^{\log_{10/7} 9/10}\, cn\right)$$



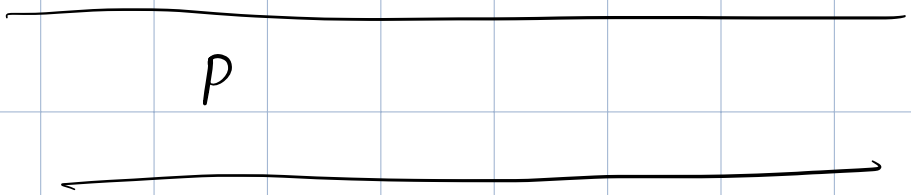

many good elements
n/3
n/3
n/3
sorted order

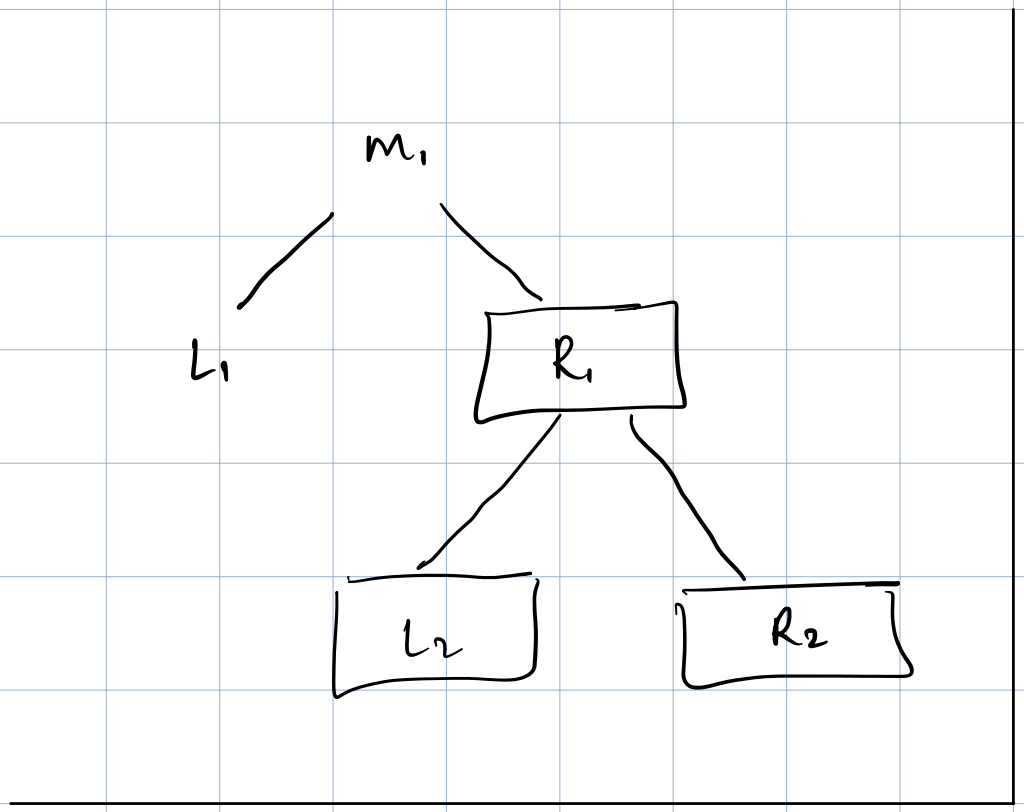



m
L
R
elements
≤ m
elements
> m

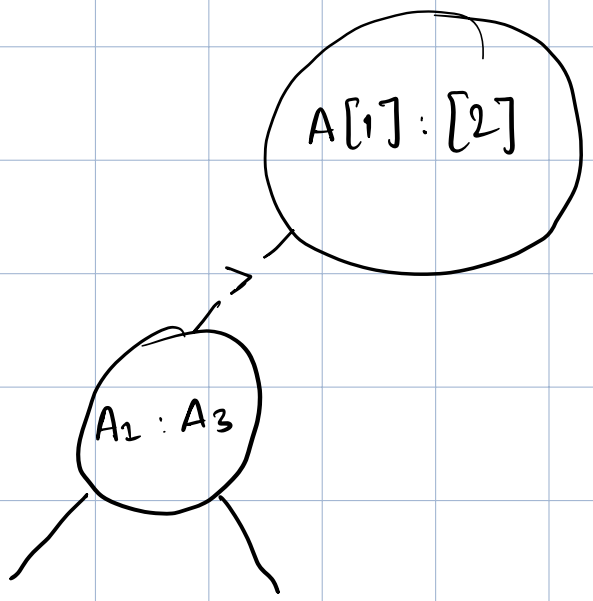
say

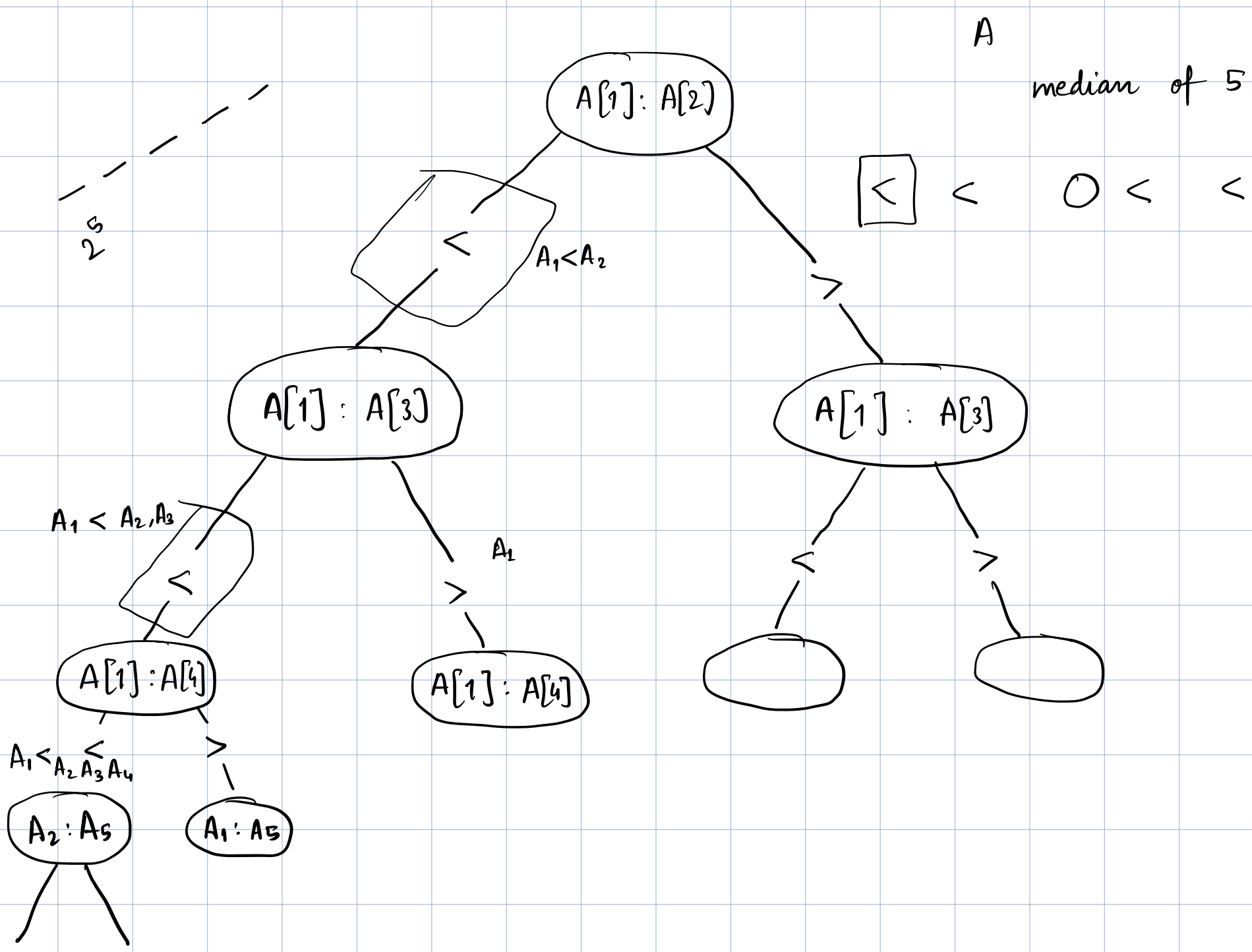
$|R| > |L|$

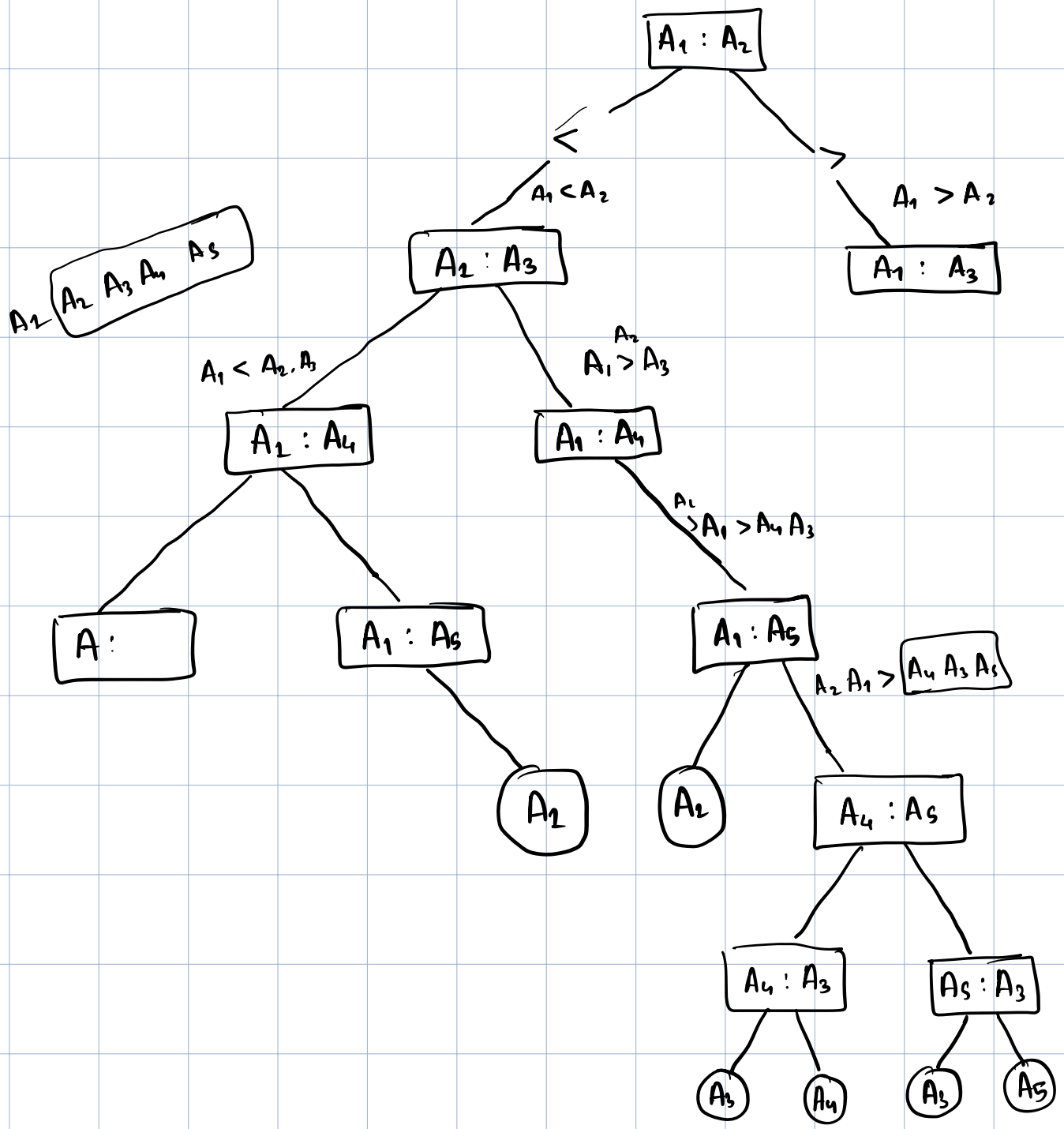

p


m_1
L_1
R_1
L_2
R_2

cu
αcu
$\alpha^2 cu$


A[1] : [2]
A1 : A3


A
median of 5
A[1] : A[2]
< < O < <
2^5
<
A1<A2
>
A[1] : A[3]
A[1] : A[3]
A1 < A2, A3
A1
<
>
<
>
A[1] : A[4]
A[1] : A[4]
A1 < A2 A3 A4
<
>
A2 : A5
A1 : A5


A1 : A2
<
A1 < A2
A1 > A2
A2 : A3
A1 : A3
A2 A3 A4 A5
A2
A1 < A2, A3
A1 > A2 A3
A2 : A4
A1 : A4
A1 > A4 A3
A1 : A5
A2 A1 > A4 A3 A5
A4 : A5
A4 : A3
A5 : A3

$X(i, j) = 1$ if A_i A_j are compared

$= 0$ o/w

$T(n) = E\left[\sum X_{(i,j)}\right]$

① local minimum in log n

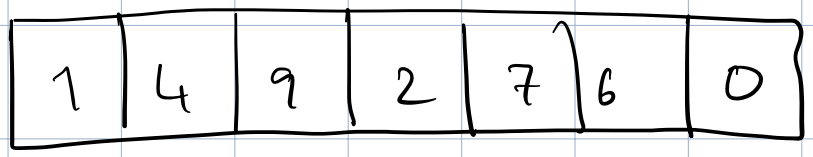
```
local_min ( A[1...n])
    if n ≤ 3
        do manual check
    else
    A = local_min ( A[1...⌊n/2⌋] )
    B = local_min ( A[⌈n/2⌉ ... n] )
    if (!A && !B) {
        check ( A[n/2 - 1] ≤ A[n/2]
                and  A[n/2] ≤ A[[n/2] + 1] )
        or
```

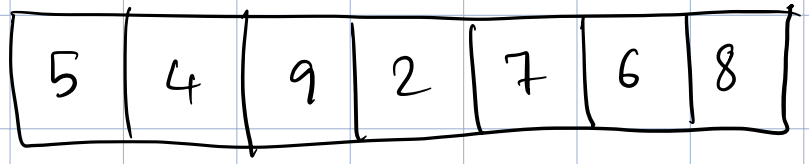
$\left(A\left\lfloor \frac{n}{2} \right\rfloor \leq A\left\lfloor \left\lfloor \frac{n}{2} \right\rfloor + 1 \right\rfloor \right.$
and $\left. A\left\lfloor \left\lfloor \frac{n}{2} \right\rfloor + 1 \right\rfloor \leq A\left\lfloor \left\lceil \frac{n}{2} \right\rceil + 2 \right\rfloor \right)$

```
else {
    return A or B (one that exists)
}
```

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + \underbrace{O(1)}_{c}$$

$$= O(n)$$


1 4 9 2 7 6 0


5 4 9 2 7 6 8